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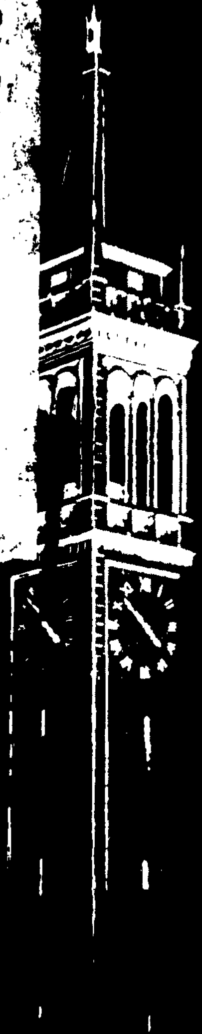

Waves in Drifting and Accelerating Electron Streams In Radial Flow

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Series No. 60, Issue No. 459

Contract No. AF 49(638)–102

June 29, 1962

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Division File No. 13-25-J

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I. INTRODUCTION

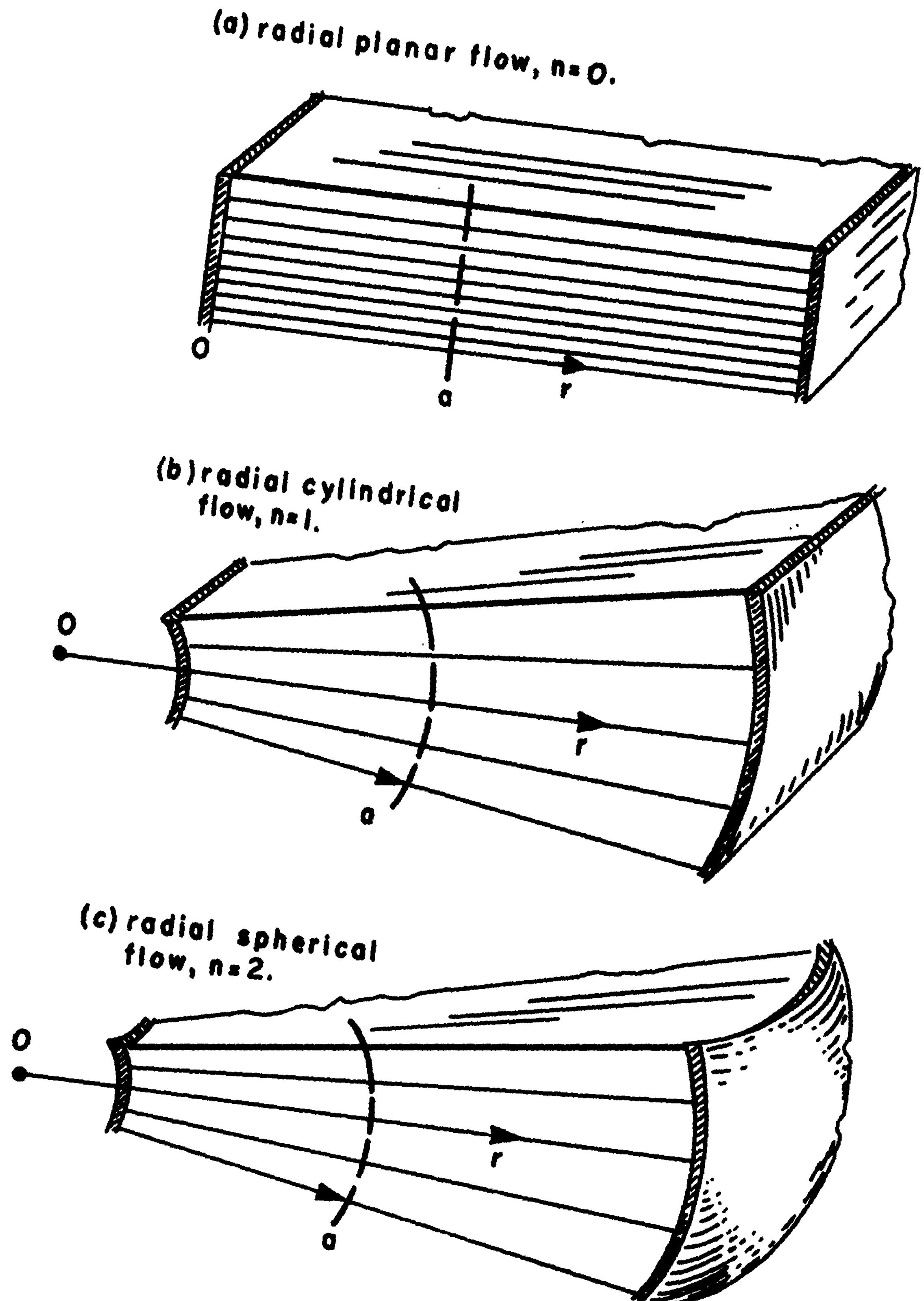
The solutions for space-charge waves obtained by Hahn (1939) and Ramo (1939) for a rectilinear, drifting stream are quite well known. Less well known are the somewhat more mathematically complicated solutions obtained when the stream flow is allowed to be along radial lines and when the stream is accelerating. First-order current for radial drifting flow has been obtained by Feenberg (1946). Current and velocity in accelerating parallel flow have been obtained by Llewellyn (1941), by Smullin (1951), Tien (1952), Hutter (1954), Müller (1955), Higuchi (1956), and Eichenbaum and Peter (1959). Accelerating radial flow was looked into by Quate (1952). The present work attempts to unify all of these cases and a few more in one general solution.

II. BASIC PROBLEM

As with parallel flow, it is desirable to know how an initial modulation is carried by the stream from one point to another. The models to be analyzed are shown in Fig. 1. The mode with motion along radius r only and with no variations of any variable normal to r will be studied. The solutions will be for $i_1(r,t)$, $v_1(r,t)$, $\rho_1(r,t)$, and $E_1(r,t)$ for all r and t.

III. GOVERNING EQUATIONS, NEW VARIABLES

The linearization used in parallel flow is applicable. Eulerian hydrodynamical equations will be used. Assuming a driving signal sinusoidal with time, the zero-order equation (dc) and the first-order (ac) equations may be separated. The dc potential V_o, and hence the dc velocity v_o, is allowed to vary with r. The velocity variation $v_o(r)$



(a) radial planar flow, n=0.
O
a
r
(b) radial cylindrical flow, n=1.
O
r
a
(c) radial spherical flow, n=2.
O
r
a

Figure 1

will be left arbitrary, to be specified later to suit specific problems. The state of the background is hidden in v_o in that the presence of ions or electrodes, or just average electron space charge is implied by the choice of $v_o(r)$. Because the flow expands (or contracts) i_o and ρ_o also depend on r.

The zero-order equations are: the r^{th} component of the equations of motion,

$$\frac{dv_o}{dt} = v_o \frac{dv_o(r)}{dr} = \eta E_o(r); \tag{1}$$

the definition of current

$$i_o = \rho_o v_o ; \tag{2}$$

and the continuity equation, for no variations normal to r,

$$\nabla \cdot i_o = \frac{1}{r^n} \frac{d}{dr}(r^n i_o) = 0 . \tag{3}$$

Here n=0,1,2 for planar, cylindrical and spherical flow respectively. The continuity equation may be integrated to obtain

$$i_o(r) = (\frac{a}{r})^n i_o(a) = (\frac{a}{r})^n i_{oa}, \tag{4}$$

where a is some conveniently chosen radius, such as the radius of the input. Quantities at radius a will be denoted by the additional subscript a. Similarly

$$\rho_o(r) = \frac{i_{oa}}{v_o(r)} (\frac{a}{r})^n \tag{5}$$

The first-order equation of motion is just the r component equation,

$$\frac{dv_1}{dt} = \eta E_1 \tag{6}$$

The definition of current is

$$i_1 = \rho_o v_1 + \rho_1 v_o \tag{7}$$

The continuity equation is

$$\nabla \cdot \underset{\sim}{i}_1 + \frac{\partial \rho_1}{dt} = 0, \tag{8}$$

which is simply

$$\frac{1}{r^n} \frac{\partial}{\partial r} (r^n i_1) + \frac{\partial \rho_1}{\partial t} = 0 \tag{9}$$

Several simplifications may be made at this stage in order to reduce the algebraic work. The reader will recall that in planar flow the stream (or mean) phase,

$$\bar{\Phi}_e = \omega (t - \frac{r}{v_o}) \tag{10}$$

occurs in the solutions for i_1, v_1, ρ_1, and E_1. Thus, we anticipate that it will also occur here, and recognize $\bar{\Phi}_e$ as

$$\bar{\Phi}_e = \omega (t - T) \tag{11}$$

where T is the zero-order transit time from the source to the point r. T is T(r) where v_o changes with distance, as

$$T(r) = \int_{\text{source}}^{r} \frac{dr}{v_o(r)} , \tag{12}$$

and

$$\frac{dT}{dr} = \frac{1}{v_o(r)} . \tag{13}$$

Inserting the phase and a judicious choice on first-order quantities, the new variables are chosen to be:

$$i(r,t) = i_o(r) + i_1(r,t) = i_o(r) + r^{-n} \hat{i}_1(r) \, e^{j\omega \left[t - T(r)\right]} , \tag{14}$$

$$v(r,t) = v_o(r) + v_1(r,t) = v_o(r) + \hat{v}_1(r) e^{j\omega \left[t - T(r)\right]} , \tag{15}$$

$$\rho(r,t) = \rho_o(r) + \rho_1(r,t) = \rho_o(r) + r^{-n} \hat{\rho}_1(r) e^{j\omega \left[t - T(r)\right]} , \tag{16}$$

$$E(r,t) = E_o(r) + E_1(r,t) = E_o(r) + r^{-n}\hat{E}_1(r)e^{j\omega\left[t-T(r)\right]}. \tag{17}$$

The units of the new variables are not fixed; for example, $\hat{i}_1$, has the units of amps/m^2 for $n=0$, amps/m for $n=1$ (to be interpreted as amps per meter length/2π along z, the cylindrical axis) and amps for $n=2$ (to be interpreted as the total current/(4π)).

Using the new variables in the first-order equations, one obtains, with exp $j\omega(t-T)$ understood,

$$v_o \frac{d\hat{v}_1}{dr} + \hat{v}_1 \frac{dv_o}{dr} = \eta r^{-n}\hat{E}_1 \tag{18}$$

Note that the $\partial v_1/\partial t$ term was cancelled by part of the $(v_o+v_1)d(v_o+v_1)/dr$ term, the dT/dr part, a saving afforded by insertion of the mean phase. The remaining equations are

$$\frac{d\hat{i}_1}{dr} - j\frac{\omega}{v_o}\hat{i}_1 + j\omega\hat{\rho}_1 = 0, \tag{19}$$

$$\hat{i}_1 = r^n \rho_o \hat{v}_1 + v_o \hat{\rho}_1 \;. \tag{20}$$

Next, the equation in $\hat{i}_1$ and $\hat{E}_1$ is to be obtained. Insert $\hat{\rho}_1$ from (20) into (19) and insert ρ_o from (5) to obtain

$$\hat{v}_1 = \frac{v_o^2}{j\omega a^n i_{oa}} \frac{d\hat{i}_1}{dr} \tag{21}$$

Differentiate this to obtain

$$\frac{d\hat{v}_1}{dr} = \frac{1}{j\omega a^n i_{oa}}\left[\left(2v_o \frac{dv_o}{dr}\frac{d\hat{i}_1}{dr}\right) + v_o^2 \frac{d^2\hat{i}_1}{dr^2}\right] \tag{22}$$

Insert $\hat{v}_1$ from (19) and $\frac{d\hat{v}_1}{dr}$ from (22) into (18) to obtain the desired equation in $\hat{i}_1$ and $\hat{E}_1$

$$\frac{d^2\hat{i}_1}{dr^2} + \left(\frac{3}{v_o}\frac{dv_o}{dr}\right)\frac{d\hat{i}_1}{dr} = \frac{\eta i_{oa}}{\epsilon_o v_o^3}\left(\frac{a}{r}\right)^n j\omega\epsilon_o E_1 \tag{23}$$

The right hand side can be simplified by recognizing that the multiplier of the displacement current is the local plasma wave number (squared) as,

$$\frac{\eta i_{oa}}{\epsilon_o v_o^3}\left(\frac{a}{r}\right)^n = \left(\frac{\eta \rho_o(r)}{\epsilon_o}\right)\frac{1}{v_o^2(r)} = \frac{\omega_p^2(r)}{v_o^2(r)} = \beta_p^2(r) \tag{24}$$

Hence (23) is

$$\frac{d^2\hat{i}_1}{dr^2} + \left(\frac{3}{v_o}\ \frac{dv_o}{dr}\right)\ \frac{d\hat{i}_1}{dr} = \beta_p^2(r)(j\,\omega\,\epsilon_o\hat{E}_1) \tag{25}$$

or, if one likes, it is

$$\frac{d^2 i_1}{dT^2} + \left(\frac{3}{v_o}\ \frac{dv_o}{dT}\right)\frac{d\hat{i}_1}{dT} = \omega_p^2(r)(j\omega\epsilon_o\hat{E}_1) \tag{26}$$

The last of the trio of field-driven equations to be obtained is the equation relating $\hat{\rho}_1$ and $\hat{E}_1$. The steps in derivation are, roughly: take v_o^2 times the definition of current, take d/dr of this, expand the derivative of $\hat{i}_1 v_o^2$, insert $d\hat{i}_1/dr$ from earlier work and $\hat{i}_1$ from the definition of current; insert this result into the $d(\hat{\rho}_1 v_o^3)/dr$ equation, factor out $\hat{v}_1 v_o$ and take d/dr of this, then insert $\eta r^{-n}\hat{E}_1$ for this derivative. The result is

$$\frac{d^2(\hat{\rho}_1 v_o)}{dr^2} + \frac{d(\hat{\rho}_1 v_o)}{dr}\left\{\frac{2}{v_o^2}\ \frac{dv_o}{dr} + i_{oa}a^n\left[\frac{1}{v_o^2}\ \frac{d^2 v_o}{dr^2}\bigg/\left(\frac{j\omega}{v_o} + \frac{2}{v_o}\ \frac{dv_o}{dr}\right)\right]\right\}$$

$$= \frac{\eta}{r^n}\hat{E}_1\left[\frac{n}{r}i_{oa}a^n + \frac{1}{v_o}\ \frac{dv_o}{dr} + \left(\frac{-\omega^2}{v_o^2} + 2j\frac{\omega}{v_o^2}\ \frac{dv_o}{dr} + \frac{2}{v_o}\ \frac{d^2 v_o}{dr^2}\right)\bigg/\left(\frac{j\omega}{v_o} + \frac{2}{v_o}\ \frac{dv_o}{dr}\right)\right]$$

$$- \eta\, i_{oa}\left(\frac{a}{r}\right)^n \frac{d\hat{E}_1}{dr} \tag{27}$$

Equations (18), (19), and (27) may be used with any form of driving field.

IV. KINETIC POWER; KINETIC ENERGY; CONSERVATION THEOREM

A simple power theorem is readily derived for the case of radial flow. The equation of motion is

$$\frac{\partial v_1}{\partial t} + v_o \frac{\partial v_1}{\partial r} + v_1 \frac{\partial v_o}{\partial r} = \eta E_1$$

or

$$\frac{\partial v_1}{\partial t} + \frac{\partial}{\partial r}(v_1 v_o) = \eta E_1$$

Multiply both sides by $(r^n i_1)/\eta$ and add and subtract $\frac{v_1 v_o}{\eta} \frac{\partial}{\partial r}(r^n i_1)$ to give

$$\frac{r^n i_1}{\eta} \frac{\partial v_1}{\partial t} + \frac{r^n i_1}{\eta} \frac{\partial (v_1 v_o)}{\partial r} + \frac{v_1 v_o}{\eta} \frac{\partial}{\partial r}(r^n i_1) - \frac{v_1 v_o}{\eta} \frac{\partial}{\partial r}(r^n i_1) - r^n i_1 E_1 = 0$$

The second and third terms combine; the fourth term can be given in terms of $\partial \rho_1/\partial t$ from the equation of continuity and then combined with the $\rho_1 v_o$ part of the first term; i_1 in the last term should be replaced by $J_1(t) - \epsilon_o \partial E_1/\partial t$ from the total current equation. Divide the result by r^n to obtain

$$\frac{1}{r^n} \frac{\partial}{\partial r}\left[r^n \left(\frac{v_o v_1 i_1}{\eta} \right) \right] + \frac{\partial}{\partial t}\left(\frac{\epsilon_o}{2} E_1^2 + \frac{\rho_o}{2\eta} v_1^2 + \frac{v_o}{\eta} \rho_1 v_1 \right) = J_1 E_1 \qquad (1)$$

The terms here may be recognized as the kinetic power density, P_k, the electric energy density, W_E, and the kinetic energy density, W_k, so that (1) may be written as,

$$\nabla_r \cdot \underset{\sim}{P}_k + \frac{\partial}{\partial t}(W_E + W_k) = J_1 E_1 \qquad (2)$$

The use of the divergence here emphasizes the meaning of conservation, that any net power density increase through a surface must be balanced by a decrease in energy density (for $J_1 = 0$) or be supplied by a source ($J_1 \neq 0$).

V. CIRCUIT EQUATION; OPEN-CIRCUIT

Where there is no ac return path the total current denisty must vanish, $J_1(t) = 0$. Hence, the open-circuit equation is

$$i_1(r,t) + \frac{\partial E_1(r,t)}{\partial t} = 0 \tag{1}$$

or

$$\hat{i}_1(r) + j\omega\epsilon_o \hat{E}_1(r) = 0$$

Inserting this value of $\hat{E}_1$ into the current equation, (III-25) produces the homogeneous equation in $\hat{i}_1$

$$\frac{d^2\hat{i}_1}{dr^2} + \left(\frac{3}{v_o}\frac{dv_o}{dr}\right)\frac{d\hat{i}_1}{dr} + \beta_p^2(r)\hat{i}_1 = 0 \tag{2}$$

This is a pleasing result from the point of view of the physics displayed; if the acceleration is mild, so that the middle term is relatively small, then $\hat{i}_1$ behaves just as in a drifting stream with simple harmonic motion at the <u>local</u> plasma frequency. It is interesting to note that this equation is formally identical with that obtained by Smullin (1951) and Tien (1952) for the case of parallel (planar) flow. The effects of radial flow are buried inside $\beta_p^2(r)$ which depends on n. It is necessary only to solve the equation in $\hat{i}_1$ because $\hat{v}_1$ and $\hat{\rho}_1$ are easily obtained from $\hat{i}_1$ using the earlier equations. It is worthwhile to give the homogeneous equation in velocity,

$$\frac{d^2\hat{v}_1}{dr^2} + \left(\frac{2}{v_o}\frac{dv_o}{dr} + \frac{n}{r}\right)\frac{d\hat{v}_1}{dr} + \left(\beta_p^2(r) + \frac{n}{rv_o}\frac{dv_o}{dr} + \frac{1}{v_o}\frac{d^2v_o}{dr^2}\right)v_1 = 0, \tag{3}$$

if only to show that the symmetry among the i_1, v_1, and ρ_1 equations obtained in planar drifting flow no longer holds.

VI. OPEN CIRCUIT SOLUTIONS FOR $v_o(r) \sim r^m$

The variation of $v_o(r)$ with r must be given in order to solve (V-2) or (V-3). Among the many possible variations of interest, a common one is that for v_o varying as some power of the radius,

$$v_o(r) = \left(\frac{r}{a}\right)^m v_{oa} , \tag{1}$$

with corresponding dc potential

$$V_o(r) = -\frac{v_o^2(r)}{2\eta} = -\frac{1}{2\eta}\left(\frac{r}{a}\right)^{2m} v_{oa}^2 = \left(\frac{r}{a}\right)^{2m} V_{oa} . \tag{2}$$

These variations have been used to solve the planar flow model for $2m = 1$ (linear potential rise) by Tien (1952), and more generally, by Müller (1955). The extension to radial flow is relatively straightforward. With v_o as given,

$$\frac{1}{v_o}\frac{dv_o}{dr} = \frac{m}{r} ,$$

$$\frac{1}{v_o}\frac{d^2v_o}{dr^2} = \frac{m(m-1)}{r^2} ,$$

$$\beta_p^2(r) = \beta_{pa}^2\left(\frac{a}{r}\right)^{3m+n} .$$

(V-2) and (V-3) then become

$$\frac{d^2\hat{i}_1}{dr^2} + \frac{3m}{r}\frac{d\hat{i}_1}{dr} + \beta_{pa}^2\left(\frac{a}{r}\right)^{3m+n}\hat{i}_1 = 0, \tag{3}$$

$$\frac{d^2\hat{v}_1}{dr^2} + \frac{2m+n}{r}\frac{d\hat{v}_1}{dr} + \left[\frac{m(m+n-1)}{r^2} + \beta_{pa}^2\left(\frac{a}{r}\right)^{3m+n}\right]\hat{v}_1 = 0. \tag{4}$$

These are Bessel equations of a standard from [see, for example, Jahnke and Emde (1945)] and have the solutions

$$\hat{i}_1 = r^{\frac{1-3m}{2}} Z_{\pm b}\left[\pm k\left(\frac{r}{a}\right)^d\right], \tag{5}$$

$$\hat{v}_1 = r^{\frac{1-(2m+n)}{2}} Z_{\pm(b+1)}\left[\pm k\left(\frac{r}{a}\right)^d\right], \tag{6}$$

where Z is a Bessel function of the first or second kind, J or N. The quantities b, d, k are given in terms of m and n by

$$b \equiv \frac{3m-1}{2-(3m+n)} \tag{7}$$

$$d \equiv \frac{2-(3m+n)}{2} \tag{8}$$

$$k \equiv \frac{2\beta_{pa}a}{2-(3m+n)} \tag{9}$$

$\hat{i}_1$ and $\hat{v}_1$ of this form satisfy (III-21) provided that like signs are chosen for both $\hat{i}_1$ and $\hat{v}_1$. The (+) sign in the argument should be taken if the stream moves in the positive r-direction (expanding stream flow) and the (-) sign if the stream moves in the negative r-direction (converging stream flow). For given values of m and n, the choice of sign of the order can be made in order to give a positive value for convenience.

It will be convenient to use a normalized radius,

$$x \equiv \frac{r}{a} \tag{10}$$

and a characteristic stream conductance,

$$g(x) \equiv \frac{\omega}{\omega_p(x)} \rho_o(x) \equiv g_a x^{-\left(\frac{n+m}{2}\right)} \tag{11}$$

with the dimensions of current density divided by velocity. The general solution for $\hat{i}_1$ is a linear combination of functions of the first and second kind as,

$$\hat{i}_1(x) = x^{-bd} \left[AJ_b(kx^d) + BN_b(kx^d) \right] \tag{12}$$

and

$$\frac{d\hat{i}_1}{dx} = -\frac{kd}{a} x^{-bd+d-1} \left[AJ_{b+1}(kx^d) + BN_{b+1}(kx^d) \right] \tag{13}$$

where A and B are arbitrary constants. From (III-21) and (1), (11), and (13) above, one obtains

$$v_1(x) = \frac{x^{\left[\frac{1-(2m+n)}{2}\right]}}{g_a a^n} \left[AJ_{b+1}(kx^d) + BN_{b+1}(kx^d) \right] \tag{14}$$

There are, of course, a variety of ways to specify boundary conditions in order to fix A and B. One way is to specify the current and velocity fluctuation at the initial plane, $i_1(a)$ and $v_1(a)$. This specification gives the solution in the form of a transmission (or "ABCD") matrix, which may be multiplied with similar matrices, allowing a cascaded solution for the more complicated dc potential distributions. One obtains

$$\begin{aligned} \hat{i}_1(x) = \frac{\pi k}{2} x^{-bd} \Big\{ & \left[J_{b+1}(k)N_b(kx^d) - N_{b+1}(k)J_b(kx^d) \right] \hat{i}_{1a} \\ & + j \left[J_b(k)N_b(kx^d) - N_b(k)J_b(kx^d) \right] a^n g_a \hat{v}_{1a} \Big\} , \end{aligned} \tag{15}$$

$$\hat{v}_1(x)a^n g(x) = \frac{\pi k}{2} x^{-(bd+n)} \Bigg\{ j \Big[J_{b+1}(k)N_{b+1}(kx^d) - N_{b+1}(k)J_{b+1}(kx^d) \Big] \hat{i}_{1a}$$

$$- \Big[J_b(k)N_{b+1}(kx^d) - N_b(k)J_{b+1}(kx^d) \Big] a^n g_a \hat{v}_{1a} \Bigg\} . \tag{16}$$

The bracketed terms of these equations are in the form of the biradial Bessel functions, defined as

$$H_{\mu,\nu}(\xi,\zeta) = \frac{\pi}{2} \Big[J_\mu(\xi)N_\nu(\zeta) - N_\mu(\xi)J_\nu(\zeta) \Big] \tag{17}$$

These and similar functions have been defined and tabulated by various authors and are useful not only as shorthand notation but for their properties of differentiation, integration, recursion, etc. [see Kino, (1955)]. Using the biradial functions in (15) and (16) and transforming back to the laboratory coordinate system, one has

$$\begin{pmatrix} i_1(x,t) \\ g(x)v_1(x,t) \end{pmatrix} = \begin{pmatrix} H_{b+1,b}(k,kx^d) & jH_{b,b}(k,kx^d) \\ jH_{b+1,b+1}(k,kx^d) & -H_{b,b+1}(k,kx^d) \end{pmatrix} \begin{pmatrix} i_{1a} \\ g_a v_{1a} \end{pmatrix}$$

$$\cdot\ kx^{d-1/2} \exp j \left[\omega t - \frac{\omega a}{v_{oa}} \ \frac{x^{1-m} - 1}{1-m} \right] \tag{18}$$

This equation is most extraordinary in that (for $v_o \sim r^m$) it covers <u>planar</u>, <u>cylindrical</u>, and <u>spherical</u> <u>streaming</u>, <u>drifting</u> or <u>accelerating</u> <u>flow</u>!

VII. REDUCTION TO PLANAR DRIFTING FLOW; $n=0$, $m=0$

For $m=0$, $n=0$, (VI-3) and (VI-4) become the planar space-charge wave equations which have solutions $\cos\beta_p r$ and $\sin\beta_p r$. It is instructive to note that (VI-18) also produces these answers. For $m=0$ and $n=0$,

$$b = -1/2$$
$$d = 1$$
$$k = \beta_{pa} a$$
$$g(x) = g_a$$

The biradial Bessel functions now have half integer orders, which are particularly simple, as

$$J_{1/2}(\xi) = \sqrt{\frac{2}{\pi}}\,\frac{\sin\xi}{\sqrt{\xi}} = N_{-1/2}(\xi)\,,$$
$$J_{-1/2}(\xi) = \sqrt{\frac{2}{\pi}}\,\frac{\cos\xi}{\sqrt{\xi}} = -N_{1/2}(\xi)\,.$$

Using these relations, (VI-18) reduces to

$$\begin{pmatrix} i_1(r,t) \\ g_a v_1(r,t) \end{pmatrix} = \begin{pmatrix} \cos\beta_{pa}(r-a) & j\sin\beta_{pa}(r-a) \\ j\sin\beta_{pa}(r-a) & \cos\beta_{pa}(r-a) \end{pmatrix} \begin{pmatrix} i_{1a} \\ g_a v_{1a} \end{pmatrix} \exp j\omega\left[\left(t-\frac{r-a}{v_{oa}}\right)\right] \tag{1}$$

VIII. FASTER AND SLOWER SPACE-CHARGE WAVE AMPLITUDES

The first space-charge wave analyses were made by Hahn (1939) and Ramo (1939) for a planar drifting stream and many of their concepts have formed a foundation for our thinking about waves in electron streams. One of these concepts is that of "faster" and "slower" space-charge waves; that is, the fluctuation amplitude in space is thought of as the interference between two constant amplitude traveling waves, traveling faster and slower than the average stream velocity. One would like to see whether this concept can be extended to the radially flowing stream and what parts of the extension are useful.

The solution for the planar drift space was given in the preceding section as

$$i_1(r,t) = \left\{\cos\left[\beta_{pa}(r-a)\right] i_{1a} + j\sin\left[\beta_{pa}(r-a)\right] g_a v_{1a}\right\} \cdot \exp\left\{j\left[\omega t - \beta_e(r-a)\right]\right\} \tag{1}$$

Writing the cosine and sine in terms of the exponentials gives

$$i_1(r,t) = \left\{\frac{1}{2}(i_{1a}+v_{1a}g_a)\exp\left[j\beta_{pa}(r-a)\right] + \frac{1}{2}(i_{1a}-v_{1a}g_a)\exp\left[-j\beta_{pa}(r-a)\right]\right\}\exp j\ \omega t-\beta_e(r-a)\ , \tag{2}$$

or

$$i_1(r,t) = \frac{1}{2}(i_{1a}+v_{1a}g_a)\exp j\left[\omega t-(\beta_e-\beta_{pa})(r-a)\right] + \frac{1}{2}(i_{1a}-v_{1a}g_a)\exp j\left[\omega t-(\beta_e+\beta_{pa})(r-a)\right]\ . \tag{3}$$

The first and second terms of (3) are called the faster and slower space-charge waves, respectively, with space-independent amplitudes and wave-numbers:

$$\begin{aligned} |i_{1f}(a)| &= \frac{1}{2}(i_{1a}+v_{1a}g_a), \quad \beta_f = \beta_e - \beta_{pa} \\ |i_{1s}(a)| &= \frac{1}{2}(i_{1a}-v_{1a}g_a), \quad \beta_s = \beta_e + \beta_{pa} \end{aligned} \tag{4}$$

The solution for radial flow with $v_o \sim r^m$ as obtained earlier is repeated here:

$$i_1(x) = (H_{b+1,b} i_{1a} + jH_{b,b} g_a v_{1a}) kx^{d-1/2} \exp\left\{j\left[\omega t - \int_1^x \beta_e(x)dx\right]\right\} \quad (5)$$

The argument of the biradial functions is (k, kx^d) and is omitted for simplicity. This equation can be rearranged to have a form similar to (2):

$$i_1(x) = \left[\frac{1}{2}(i_{1a}+v_{1a}g_a)(H_{b+1,b}+jH_{b,b}) + \frac{1}{2}(i_{1a}-v_{1a}g_a)(H_{b+1,b}-jH_{b,b})\right] kx^{d-1/2}$$

$$\cdot \exp\left\{j\left[\omega t - \int_1^x \beta_e(x)dx\right]\right\} \quad (6)$$

The combinations of biradial functions occurring in (6) become exponentials for large argument. For all arguments, these combinations may be written as exponentials times a varying magnitude, as

$$H_{b+1,b} \pm jH_{b,b} = M^i(x) \exp\left[\pm j\phi^i(x)\right] \quad (7)$$

where

$$M^i(x)^2 \equiv H^2_{b+1,b} + H^2_{b,b}$$

$$\phi^i(x) \equiv \tan^{-1}\left(\frac{H_{b,b}}{H_{b+1,b}}\right) \quad (8)$$

From the phase an effective local plasma wave number may be defined, for current, as

$$\left[\beta_p^i(x)\right]_{eff} \equiv \frac{1}{a} \frac{d}{dx}\left[\tan^{-1}\left(\frac{H_{b,b}}{H_{b+1,b}}\right)\right]$$

$$= \beta_p(x) \cdot \left[\frac{1}{(k^2 x^d)(H^2_{b,b}+H^2_{b+1,b})}\right] \quad (9)$$

The bracketed term in (9) is unity for sufficiently large arguments. The faster and slower waves thus defined have non-constant amplitude:

$$|i_{1f}(x)| = \frac{1}{2}(i_{1a} + g_a v_{1a}) M^i(x), \quad \beta_f(x) = \beta_e(x) - \left[\beta_p^i(x)\right]_{eff}$$

$$|i_{1s}(x)| = \frac{1}{2}(i_{1a} - g_a v_{1a}) M^i(x), \quad \beta_s(x) = \beta_e(x) + \left[\beta_p^i(x)\right]_{eff} \tag{10}$$

The same method applied to the velocity equation yields a <u>different</u> effective local plasma wave number:

$$\begin{aligned} \left[\beta_p^v(x)\right]_{eff} &= \frac{1}{a}\frac{d}{dx}\left[\tan^{-1}\left(\frac{H_{b+1,\,b+1}}{-H_{b,\,b+1}}\right)\right] \\ &= \beta_p(x)\left[\frac{1}{(k^2x^d)(H^2_{b+1,\,b+1}+H^2_{b,\,b+1})}\right] \end{aligned} \tag{11}$$

Again the bracketed term is unity for sufficiently large arguments.

Using the kinetic power density as calculated earlier,

$$\overline{P}_k \equiv \mathrm{Re}\left[\frac{v_o(x) i_1(x) v_1^*(x)}{\eta}\right] \tag{12}$$

it can be shown that

$$x^n \overline{P}_k(x) = \overline{P}_{ka} = x^n \overline{P}_{kf}(x) + x^n \overline{P}_{ks}(x) = \overline{P}_{kfa} + \overline{P}_{ksa} \tag{13}$$

where

$$\overline{P}_{kf}(x) = \mathrm{Re}\left[\frac{v_o(x)\, i_{1f}(x)\, v_{1f}^*(x)}{\eta}\right] \tag{14}$$

and similarly for the slower wave. Not only is the total kinetic power conserved in radial flow, but the kinetic power in the faster and slower waves defined by (10) is conserved separately, similar to the results for planar flow.

IX. SOLUTIONS FOR $3m+n = 2$

Using $3m+n = 2$ causes $b \longrightarrow \infty$, $d \longrightarrow 0$, $k \longrightarrow \infty$, making awkward the use of the Bessel function solutions. Such combinations of m and n include:

(i) Planar space-charge limited flow, $m = 2/3$, $n = 0$;

(ii) Cylindrical flow $v_o \sim r^{1/3}$, $m = 1/3$, $n = 1$;

(iii) Spherical, drifting stream, $m = 0$, $n = 2$.

Although some limiting process might be used to adapt the Bessel function solutions to fit these situations, a direct solution of the current equation (VI-3) can be obtained. Inserting 2-n for 3m, one obtains,

$$\frac{d^2\hat{i}_1}{dr^2} + \frac{2-n}{r}\frac{d\hat{i}_1}{dr} + \frac{\beta_{pa}^2 a^2}{r^2}\hat{i}_1 = 0 \tag{1}$$

The solution to this equation is of the form

$$\hat{i}_1(r) \sim r^p$$

with the determinantal equation

$$p^2 + (1-n)p + (\beta_{pa} a)^2 = 0 \tag{2}$$

This equation has two solutions,

$$p = \frac{n-1}{2} \pm \sqrt{(\frac{n-1}{2})^2 - (\beta_{pa} a)^2} = \frac{n-1}{2} \pm s, \tag{3}$$

where the quantity s has been defined in an obvious way. Then one has

$$i_1(r) = A' r^{(\frac{n-1}{2} + s)} + B' r^{(\frac{n-1}{2} - s)} \tag{4}$$

$$\hat{v}_1(r) = \frac{v_o^2(r)}{j\omega i_{oa} a^n}\left[A'\left(\frac{n-1}{2}+s\right) r^{\left(\frac{n-3}{2}+s\right)} + B'\left(\frac{n-1}{2}-s\right) r^{\left(\frac{n-3}{2}-s\right)}\right] \tag{5}$$

Imposing the initial conditions at radius a and transforming back to laboratory coordinate system gives the solution

$$\begin{pmatrix} i_1(x,t) \\ g(x)\, v_1(x,t) \end{pmatrix} = \begin{pmatrix} \left[\frac{x^s+x^{-s}}{2} + \frac{1-n}{2s}\,\frac{x^s-x^{-s}}{2}\right] & j\left[\sqrt{\frac{1-n^2}{2s}-1}\;\frac{x^s-x^{-s}}{2}\right] \\ j\left[\sqrt{\frac{1-n^2}{2s}-1}\;\frac{x^s-x^{-s}}{2}\right] & \left[\frac{x^s+x^{-s}}{2} - \frac{1-n}{2s}\,\frac{x^s-x^{-s}}{2}\right] \end{pmatrix}$$

$$\cdot \begin{pmatrix} i_{1a} \\ g_a v_{1a} \end{pmatrix} x^{-\left(\frac{n+1}{2}\right)} \exp j\left[\omega t - j\,\frac{\omega a}{v_{oa}}\;\frac{3\left(x^{\frac{1+n}{3}}-1\right)}{1+n}\right] \tag{6}$$

This is obviously not an easy form to use. Other forms can be given that are simpler and also display the physical behavior more clearly.

The transmission matrix can be made simpler by looking at the nature of s. s will be real or imaginary depending on the magnitude of $(\beta_{pa} a)^2$, that is, on the current.

For planar flow, $n=0$, m is $2/3$, which is fully space-charge limited accelerated flow. This flow requires that

$$\frac{-J_a a^2}{V_a^{3/2}} = \frac{4}{9}\,\epsilon_o\sqrt{-2\eta} \tag{7}$$

The quantity $(\beta_{pa} a)^2$ is given by

$$(\beta_{pa} a)^2 = \frac{\eta}{\epsilon_o}\,\frac{J_a}{v_o^3(a)}\,a^2 = \left(\frac{-J_a a^2}{V_a^{3/2}}\right)\left(\frac{1}{2\epsilon_o\sqrt{-2\eta}}\right) \tag{8}$$

Inserting (7) into (8) leads to

$$(\beta_{pa} a)^2 = 2/9 \tag{9}$$

so that

$$s = 1/6, \; p = -1/2 \pm 1/6 = -2/3, \; -1/3 \tag{10}$$

With s real, the transmission matrix elements A, B, C, D, are

$$A = \cosh (s \ln x) + \frac{(1-n)}{2} \frac{\sinh (s \ln x)}{s} ; \tag{11a}$$

$$B = j\beta_{pa} a \frac{\sinh (s \ln x)}{s} ; \tag{11b}$$

$$C = B ; \tag{11c}$$

$$D = \cosh (s \ln x) - \frac{(1-n)}{2} \frac{\sinh (s \ln x)}{s} \tag{11d}$$

It is seen that these elements tend to increase or decrease monotonically with distance x which is quite different from the periodic elements of oridinary space-charge wave flow.

For cylindrical flow, n=1, m is 1/3 and the flow is accelerating. n-1 is zero so that s is $j\beta_{pa}a$, always imaginary. With s imaginary, the transmission matrix elements A, B, C, D, are using $s=j\zeta$

$$A = \cos(\zeta \ln x) - \frac{(1-n)}{2} \frac{\sin(\zeta \ln x)}{\zeta} ; \tag{12a}$$

$$B = -j\beta_{pa} a \frac{\sin(\zeta \ln x)}{\zeta} ; \tag{12b}$$

$$C = B ; \tag{12c}$$

$$D = \cos(\zeta \ln x) \mp \frac{(1-n)}{2} \sin\frac{(\zeta \ln x)}{\zeta} .$$

These elements, with the trigonometric functions, are periodic with x; however, because of the logarithmic dependence the zeros of the trigonometic functions may be very far apart, especially for small ζ, small currents.

For spherical flow n=2, m is zero meaning that the stream is drifting at constant velocity. Two possibilities arise here:

(i) if $\beta_{pa} a < 0.5$ (small currents) then s is real and the monotonically-increasing matrix elements of (11) are to be used;

(ii) if $\beta_{pa} a > 0.5$ (large currents) then s is imaginary and the periodic matrix elements of (12) are to be used.

The turning point is at $\beta_{pa} a$ of 0.5 which correpsonds to a critical current. At less than critical current, the behavior is similar to that of planar drifting flow in which space-charge forces have been neglected; the apparent reason is that the flow expands sufficiently rapidly so that no debunching takes place. The critical current occurs where the space-charge wavelength, λ_{pa}, is twice the circumference of the $\underline{a}$ spere. At larger currents the space-charge forces have become sufficiently strong to cause debunching before the stream has expanded appreciably. The critical current can also be associated with a perveance, as one can write $(\beta_{pa} a)^2$ in terms of the total average current I_o as,

$$(\beta_{pa} a)^2 = \frac{-I_o}{V_a^{3/2}} \frac{1}{8\pi\epsilon_o \sqrt{2\eta}} \tag{13}$$

Using the charge-to-mass ratio of electrons, the critical perveance is given by

$$\frac{I_o}{V_a^{3/2}} = -33\times10^{-6} \qquad\qquad (\text{amps/volts}^{3/2}) \qquad (14)$$

(For example, this perveance corresponds to 33μ A at 1 volt, or 33 mA at 100 volts or 33A at 10 kilovolts.) For a model in which only a sector of spherical flow is used, the critical current and perveance will be correspondingly smaller by the ratio of the area used to that of a complete sphere.

In the limit of large radius of curvature, with small spacing between input and output and at larger than critical perveance, it can be shown that the transmission matrix elements are the same as in parallel flow, ordinary space-charge waves.

X. EXAMPLE OF CYLINDRICAL DRIFTING FLOW n = 1, m = 0

The example of a cylindrical klystron is chosen to illustrate both the physical and mathematical aspects of the rather involved solutions obtained. A sketch of such a device is shown in Fig. 2. For this example, n=1, m=0, which fixes

$$b = -1;$$
$$d = 1/2;$$
$$k = 2\beta_{pa} a$$
$$g(x) = g_a / \sqrt{x}.$$

For the sake of convenience, as there is no critical current or perveance, $\beta_{pa} a$ may be chosen arbitrarily; one easy choice is to make $\beta_{pa} a = 1/2$ and thus, k=1. The relation between input and output quantities from (VI-18) is,

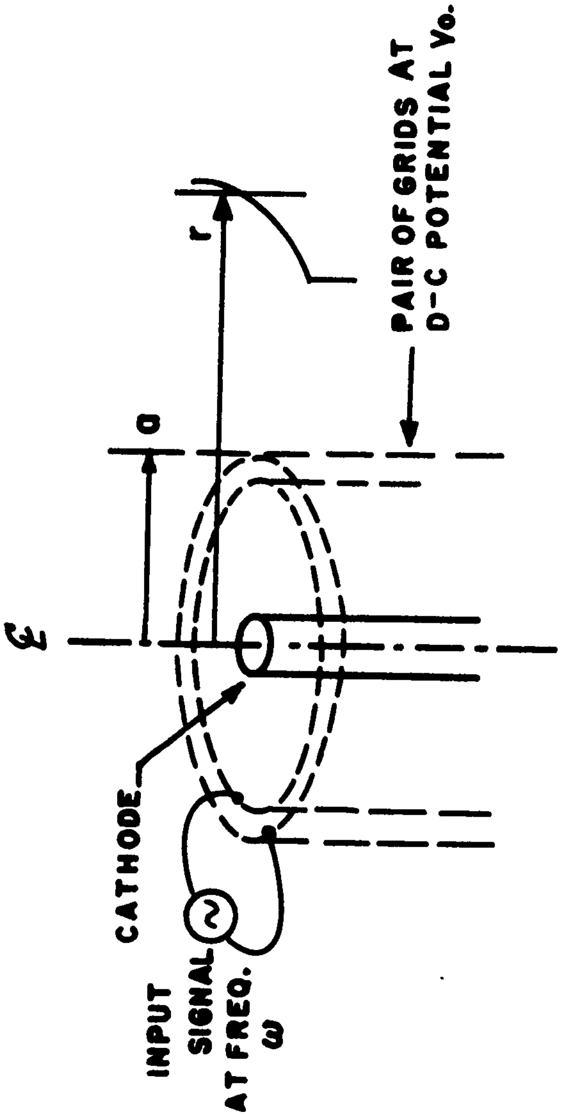

PAIR OF GRIDS AT
D-C POTENTIAL Vo.
r
a
℄
CATHODE
INPUT
SIGNAL
AT FREQ.
ω

Figure 2

$$\begin{pmatrix} i_1(x) \\ \\ g(x)v_1(x) \end{pmatrix} = \begin{pmatrix} -H_{o,1}(1,\sqrt{x}) & jH_{1,1}(1,\sqrt{x}) \\ \\ jH_{o,o}(1,\sqrt{x}) & H_{1,o}(1,\sqrt{x}) \end{pmatrix} \begin{pmatrix} i_{1a} \\ \\ g_a v_{1a} \end{pmatrix} \exp\left[-j(x-1)\right] . \tag{1}$$

(Use has been made of the recursion relations among the $H_{\mu,\nu}$ functions.)

For velocity modulation only at the a (initial) plane ($i_{1a}=0$), one has

$$\frac{v_1(x)}{v_{1a}} \frac{g(x)}{g_a} = \frac{1}{\sqrt{x}} \frac{v_1(x)}{v_{1a}} = jH_{1,o}(1,\sqrt{x}) , \tag{2}$$

and

$$\frac{i_1(x)}{g_a v_{1a}} = H_{1,1}(1, \sqrt{x}) . \tag{3}$$

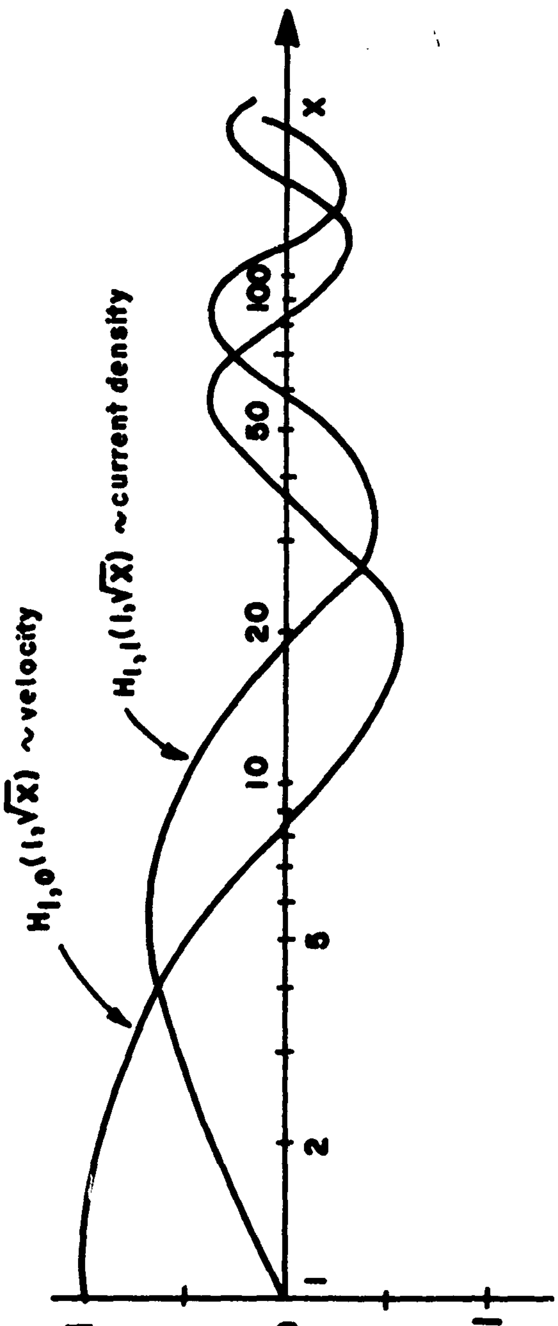
The pertinent biradial functions are shown in Fig. 3 as a function of x. Note that the velocity and current density are periodic as with planar drifting flow, but here the velocity maxima and current minima are not quite aligned, except at large distances. The bunching and debunching is much the same as in ordinary parallel-flow klystrons. However, the total ac current maxima will not stay constant but will grow with the distance as can be shown using the asymptotic formula for $H_{\mu,\nu}$. For $\xi, \zeta \longrightarrow \infty$

$$H_{\mu,\nu}(\xi,\zeta) \cong \frac{\sin\left((\zeta-\xi) - \frac{\mu-\nu}{2}\pi\right)}{\sqrt{\xi\zeta}} \quad ; \tag{4}$$

Thus, for large values of k and $k\sqrt{x}$, the current ($\sim i_1(x)\, x$) is,

$$\frac{i_1(x)\, x}{g_a v_{1a}} = \frac{\sin(k\sqrt{x} - k)}{\sqrt[4]{x}}\, x \tag{5}$$

so that the current maxima growth as $x^{3/4}$ for expanding flow.



$H_{1,0}(1,\sqrt{X})$ ~velocity
$H_{1,1}(1,\sqrt{X})$ ~current density
x
1
0
-1
1
2
5
10
20
50
100

Figure 3

For current modulation only at the a (initial) plane ($v_{ia} = 0$), one has

$$v_1(x)\,\frac{g(x)}{i_{1a}} = \frac{v_1(x)}{\sqrt{x}}\;\frac{g_a}{v_{1a}} = jH_{o,o}(1, \sqrt{x})\;, \tag{6}$$

and

$$\frac{i_1(x)}{i_{1a}} = -H_{o,1}(1, \sqrt{x})\;. \tag{7}$$

The pertinent biradial functions are shown in Fig. 4. The same comments apply as in the case of velocity modulation.

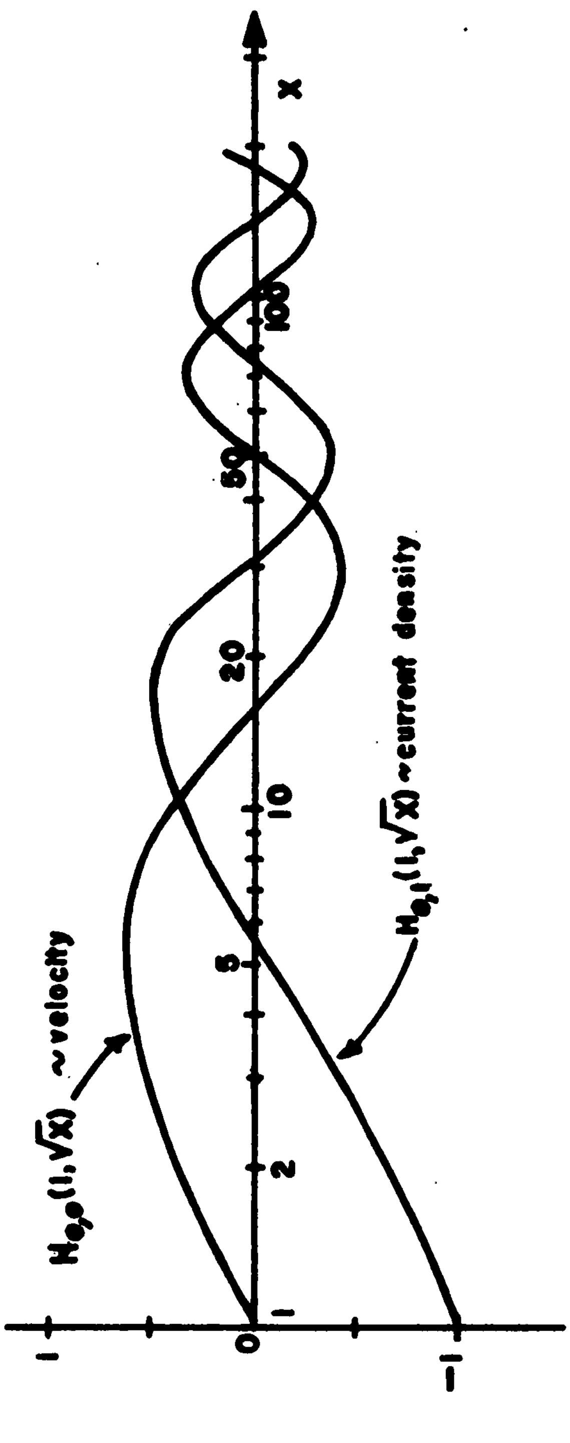
An interesting singularity in the charge density occurs under certain conditions. Recall from planar drifting flow, with only the faster wave excited, that the ac charge density vanishes when excited at $\omega = \omega_p$. In the cylindrical drifting flow with the stream expanding, the local plasma frequency decreases with increasing x. Thus, by modulating at $r = a$ with a frequency $\omega < \omega_{pa}$ and letting the flow expand, there will be a radius at which $\omega = \omega_p(r)$. The question is whether the ac charge density can be made to vanish at this radius. For the model at hand, with $n = 1$, $m = 0$, the charge density is given by

$$\frac{\hat{\rho}_1(x)}{\rho_{oa}} = \frac{\hat{i}_1(x)}{i_{oa}} - \frac{\hat{v}_1(x)}{v_{oa}}\,xa \tag{8}$$

Suppose that at some radius x_c

$$i_1(x_c) = v_1(x_c)g(x_c) \tag{9}$$

which says that the velocity and current are in phase at this point, and implies the existence of a "faster wave" only at this point, Using this



X
1
2
5
10
20
50
100
1
0
-1
$H_{e,0}(1,\sqrt{X})$ ~velocity
$H_{e,1}(1,\sqrt{X})$ ~current density

Figure 4

value for the velocity, the charge density becomes, at x_c,

$$\frac{\hat{\rho}_1(x_c)}{\rho_{oa}} = \frac{\hat{i}_1(x_c)}{i_{oa}} \left(1 - \frac{\omega_p(x_c)}{\omega} x_c\right) \tag{10}$$

Hence, the charge density can be made to vanish at the point x_c by choosing

$$\omega = \omega_p(x_c) x_c \tag{11}$$

This zero in charge density comes about not by wave interference but by a local physical requirement of zero charge density. In order for the modulation at x_c to be that specified by (9), the modulation at the $\underline{a}$ (initial) plane must be

$$i_{1a} = v_{1a} g_{1a} \left[\frac{-H_{1,o}(1, \sqrt{x_c}) + jH_{1,1}(1, \sqrt{x_c})}{H_{o,1}(1, \sqrt{x_c}) + jH_{o,o}(1, \sqrt{x_c})} \right] \tag{12}$$

The bracketed term is unity both for $x = 1$, and large values of x and tends to have magnitude near unity for values in between; thus, requiring the "faster wave" only at $x = x_c$ implies that something like the "faster wave" exists for all other x.

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